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ANALYTICAL TOOLS FOR CLOCKS IN SPACE

Scott Francis,[1] Samuel Stein,[2] Thomas Celano,[2] Jeremy Warriner,[2]
Jesse Leitner,[3] Michael Moreau,[3] Richard Burns,[3]
Robert A. Nelson,[4] and Al Gifford[5]

[1]Zeta Associates, Fairfax, VA
[2]Timing Solutions Corporation, Boulder, CO
[3]NASA Goddard Space Flight Center, Greenbelt, MD
[4]Satellite Engineering Research Corporation, Bethesda, MD
[5]NASA Headquarters, Washington, DC

Abstract - **This paper summarizes the development of mathematical simulators for precise time transfer and the underlying theoretical principles. Experimental tests with atomic clocks flown on aircraft and potential applications to future space missions are described.**
***Keywords* - Simulator, atomic clock, time transfer, relativity.**

I. INTRODUCTION

Space-qualified cesium beam and passive hydrogen maser atomic clocks routinely maintain time to a precision of 10^{-14} or better, while hydrogen masers in the laboratory offer precisions of 10^{-15} or 10^{-16}. New technologies under development for clocks in space, such as laser-cooled atomic clocks, may provide precisions approaching 10^{-17}. At this level of precision, time could be measured at the sub-picosecond level over a few hours.

In parallel to clock technology, there must be a corresponding development of theoretical methods of time dissemination within an adopted space-time coordinate system. These methods involve considerations of hardware biases, environmental effects, propagation delays, orbital perturbations, and relativity physics. To study the sensitivity of time dissemination due to these various factors, a mathematical simulator has been formulated to model relativistic time transfer among earth stations, aircraft, and satellites. In addition, the time transfer simulator has been integrated into the Formation Flying Testbed at the NASA Goddard Space Flight Center.

The simulator has been applied initially to a series of aircraft flight tests [1]. The first series of tests was staged from Edwards AFB to support the Advanced Tactical Targeting Technology (AT3) program. During each flight, the time recovered from a GPS receiver was compared with a high performance cesium atomic clock using a time-interval analyzer. The flying clock was also compared to a similar ground clock both before and after the flight. Closure between pre-flight and post-flight data would not have been obtained within the margin of experimental error had not appropriate relativistic corrections for gravitational potential, time dilation, and the Sagnac effect been included. A second series of tests was conducted from the Wright-Patterson AFB involving a C-135 aircraft using continuous Two-Way Satellite Time Transfer.

These simulations have been designed as analytical tools in support of a technology foundation for timekeeping by clocks in space. Potential applications for Earth-orbiting spacecraft include studies of the time-keeping environments of the International Space Station, highly inclined elliptical orbit satellites, and geostationary satellites. Future studies might include synchronization methods for clocks on the Moon or on Mars, with possible applications for Very Long Baseline Interferometry (VLBI), interplanetary radio-navigation references, and refined tests of general relativity.

II. TIMING REQUIREMENTS

For time measurement and time transfer, there are two equally important development activities. First, there is research in advances in clock technologies. Second, there is the analysis of theoretical algorithms and modeling. Both of these activities are necessary for achieving greater precision of coherent timekeeping and maintaining interoperability across a network of clocks.

Theoretical algorithms must consider hardware biases, environmental effects, and propagation delays. For clocks onboard spacecraft, orbital ephemerides must be generated and the effects of orbital perturbations must be included. In addition, theoretical corrections according to the special and general theories of relativity must be analyzed.

In the design of a system using precise time, the requirements may not be known *a priori*. Sources of error must be identified and corrected and their magnitudes must be identified. Thus mathematical modeling is necessary to establish and validate requirements.

III. RELATIVISTIC EFFECTS

Relativistic time transfer algorithms are necessary for high precision time transfer and dissemination [2]. For relativistic time transfer, the distinction between coordinate time and proper time must be recognized and a common space-time coordinate system must be adopted.

The Global Positioning System (GPS) has served as a laboratory for relativity physics and has provided a model for theoretical algorithms [3]. There are three principal relativistic effects. First, there is time dilation. On account of its orbital velocity of 3.874 km/s, a satellite clock runs slow

TABLE 1
Relativistic Effects on Satellite Clocks and Signal Propagation

Satellite orbital properties									
Satellite		**ISS**	**TOPEX**	**Glonass**	**GPS**	**Galileo**	**Molniya**	**GEO**	**Tundra**
Semimajor axis	km	6766	7715	25510	26562	29994	26562	42164	42164
Eccentricity		0.01	0.001	0.02	0.02	0.02	0.722	0.01	0.2684
Inclination	deg	51.6	66.0	64.8	55.0	56.0	63.4	0.1	63.4
Argument of perigee	deg	0	0	0	0	0	250	0	270
Apogee altitude	km	456	1345	19642	20715	24216	39362	36208	47103
Perigee altitude	km	320	1329	18622	19652	23016	1006	35364	24469
Ascending node altitude	km	320	1329	18622	19652	23016	10507	35364	32748
Period of revolution	s	5539	6744	40549	43082	51697	43083	86164	86164
Mean motion	rev/d	15.6	12.8	2.1	2.0	1.7	2.0	1.0	1.0
Mean velocity	km/s	7.675	7.188	3.953	3.874	3.645	3.874	3.075	3.075
Clocks									
Secular time dilation	µs/d	− 28.2	− 24.7	− 7.4	− 7.1	− 6.3	− 7.1	− 4.4	− 4.4
Secular redshift	µs/d	3.5	10.4	45.1	45.7	47.3	45.7	51.0	51.0
Net secular effect	µs/d	− 24.7	− 14.3	37.7	38.6	41.1	38.6	46.6	46.6
Periodic eccentricity effect	ns	12	1	45	46	49	1653	29	774
Secular oblateness redshift	ns/d	2.1	− 4.6	− 0.1	0.0	0.0	− 0.1	0.1	0.0
Periodic oblateness redshift	ps	171	191	31	24	21	30	0	14
Periodic tidal effect of Moon	ps	0.0	0.0	1.0	1.2	1.8	1.2	6.1	6.1
Periodic tidal effect of Sun	ps	0.0	0.0	0.5	0.5	0.8	0.5	2.7	2.7
Signal propagation									
Maximum Sagnac effect	ns	13	23	131	136	155	234	218	275
Gravitational delay along radius	ps	0.8	2.5	− 3.5	− 4.7	− 9.1	− 4.7	− 27.3	− 27.3
Periodic fractional Doppler shift	10^{-12}	13.1	1.1	7.0	6.7	5.9	241.1	2.1	56.5

relative to a clock on the geoid at a rate of 7.1 µs per day. Second, there is the gravitational redshift. At an altitude of 20 184 km, a satellite clock runs fast relative to a clock on the geoid at a rate of 45.7 µs per day. The resulting net secular effect is that the clock runs fast by 38.6 µs per day. To compensate for this drift, a GPS clock is offset prior to launch from the nominal frequency of exactly 10.23 MHz to 10 229 999.995 4326 Hz, so that the frequency of the clock when in orbit appears to have the nominal rate as observed on the Earth. However, due to the slight orbital eccentricity, the orbits are not truly circular and the velocity and altitude are not constant. Thus there is a residual periodic effect with an amplitude of about 46 ns. In addition, because a receiver at rest on the rotating Earth moves through inertial space during the time of signal propagation, there is a Sagnac effect whose maximum value for a circular orbit is 133 ns.

Corresponding corrections must be applied to clocks on the International Space Station (ISS). The ISS has a velocity of 7.675 km/s in Low Earth Orbit at an altitude of 388 km. The secular effects due to time dilation and redshift are − 28.2 µs per day and + 3.5 µs per day, respectively. The residual periodic effect is of the order 12 ns and the maximum Sagnac effect is 13 ns. Using cesium, rubidium, or laser-cooled clocks with measurements at the nanosecond to picosecond level of precision, it is necessary to model the transformation of proper time to coordinate time. The fine details of the orbital perturbations must be taken into account, such as by using real time measurements with GPS and long term analytical predictions. The orbital perturbations include those due to the zonal and tesseral harmonics of the Earth's potential, the gravitational attractions of the Sun and Moon, solar radiation pressure, and atmospheric drag. The effect of drag on the Station's large cross section alone is to cause the orbit to decay at an average rate of 0.2 km per day. Relativistic corrections for clocks and signal propagation for various satellite orbits are summarized in *Table 1*.

Synchronization between clocks on Earth and Mars present additional considerations. The time coordinate for relativistic ephemerides as given by the proper time readings of clocks on the geoid is Terrestrial Time (TT). A practical realization of TT in terms of International Atomic Time (TAI) is TT = TAI + 32.184 s [4]. The timescale for planetary motion in the heliocentric frame of reference is Barycentric Coordinate Time (TCB). There is a relativistic transformation between TT and TCB. An analogous transformation exists between proper time on Mars and TCB. As the semimajor axis of the orbit of Mars is 1.524 AU, the propagation time for an electromagnetic signal is between 4.4 minutes and 21.0 minutes. The round trip relativistic time delay due to the solar gravitational potential is 240 µs at superior conjunction.

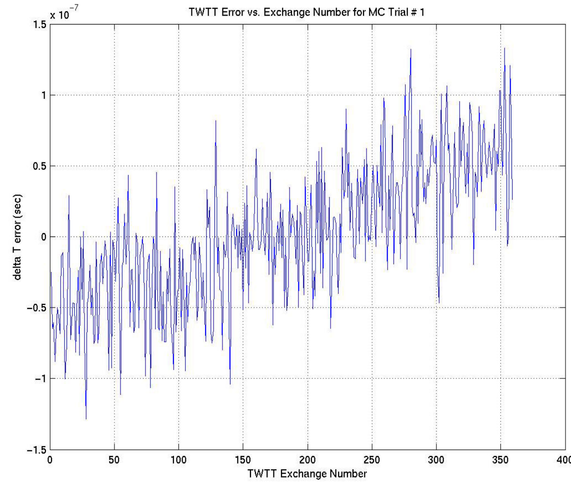
IV. SATELLITE CLOCK SIMULATIONS

To study the theoretical importance of various hardware-related biases, environmental factors, propagation delays, orbital perturbations, and relativistic effects on satellite clocks, a Two-Way Time Transfer Simulator has been developed as a MATLAB™-based tool that simulates a series of measured clock differences between moving platforms. Exchanges directly between two platforms or via a relay are supported. Noise contributions from a typical set of hardware components have been modeled, including special modules for clocks and modems. The platforms may have an arbitrary relative motion.

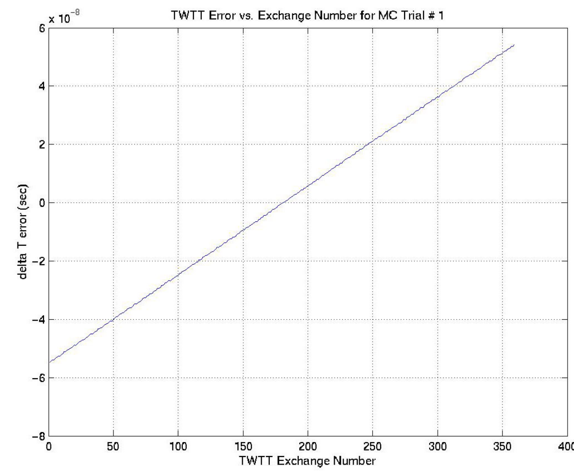
The simulation conducts a series of signal exchanges between the two platforms. Each platform transmits a timing signal according to a local clock's on-time marker (OTM). The simulator performs the calculations necessary to transform this local "hardware" proper time to an "ideal" proper time by correcting for the hardware biases. Then the coordinate time in an ECI reference frame is computed using a relativistic transformation. Based on this coordinate time of transmission, the simulator searches for a solution to the time of fight between the platforms. At the receiving platform, the calculations are inverted to compute the ideal proper time and the hardware proper time of reception. Finally, the platforms exchange their respective measurements of the difference between the local OTM and the received OTM, which are combined to determine the proper time difference between the clocks. The key features of the simulation are its inclusion of relativistic effects, hardware noise, and arbitrary platform motion during the exchange.

Models for clock, modem, and measurement noise are implemented directly in the code. The simulator supports future modules to include variations in propagation delay. The clock noise is generated from a third-order model that includes noise densities for white phase noise, white frequency noise, frequency aging, and random-walk frequency aging. Noise densities for a high performance cesium standard are used in the simulations presented in this paper. The algorithm itself can accept clock steering inputs during the simulation. Modem noise is generated from a model that represents the performance of a commercially available satellite modem with special modifications to transmit low-latency OTM signals. Noise associated with the timer is based on the performance of a high quality two-channel timer. Platform motion may be specified in several ways: directly via a set of waypoints that are time-stamped in hardware proper time, by a set of orbital elements, or generated algorithmically by a user-specified function.

Figures 1a and *1b* show the simulation results for a fixed platform located at the National Institute for Standards and Technology (NIST) laboratories in Boulder, Colorado and a moving platform in an orbit corresponding the International Space Station. *Figure 1(a)* includes the effects of modem noise, while *Figure 1(b)* shows the results when the modem noise is not included. The simulation time coincides with the portion of the orbit that is line-of-sight visible to NIST. Since the relative drift between the clocks themselves is removed from these plots, the resulting data show only the relativistic effects associated with the platform kinematics and gravitational potential and the noise introduced by the measurement and signal generation hardware. Analogous simulations were performed for a highly inclined, elliptical Molniya orbit and a geostationary orbit. *Table 2* summarizes the range of errors for these simulations.



(a) modem noise included



(b) modem noise not included

Figure 1. Error between actual and measured clock offsets for time transfers between NIST and the ISS.

TABLE 2
Error Ranges for Three Orbits

Orbit	Error Range
ISS	− 55 ns to + 53 ns
Molniya	− 96 ns to + 270 ns
Geostationary	+ 2.8 ns to + 3.3 ns

V. MODELING OF RELATIVISTIC EFFECTS ON MOVING CLOCKS

It is well known that special and general relativity cause clocks to run either fast or slow compared to clocks at rest in the Earth's rotating reference frame. These effects, as discussed above, are explicitly modeled in the TWTT simulator. Recent C-135 test flights staged from Wright-Patterson AFB during November, 2002 provided an opportunity to validate our relativity modeling. These experiments represent a significant step in a deliberate, iterative approach toward modeling and experimentally validating time transfer techniques for clocks in space.

A series of five flight tests were flown to evaluate Two-Way Time Transfer between a ground station and an airborne platform using both a direct line-of-sight radio-frequency (RF) link as well as a geostationary satellite communications relay. A companion paper [5] describes these flights in greater detail and provides results of the experimental procedures and measurements. Our focus here is on the relativistic effects on the flight clock compared to a clock that remained on the ground (both were high performance cesium standards). These flight tests were invaluable, since we could measure clock differences throughout the flight rather than just prior to and after the flight.

Relativistic effects are accounted for by the transformation between proper time and coordinate time in a rotating Earth-Centered Earth-Fixed (ECEF) coordinate system. The simulator includes the corrections for time dilation, gravitational redshift, and the Sagnac effect, as given by the formulas

$$\Delta T_v = -\frac{1}{2c^2}\sum_{i=1}^{N} v_i^2\,\Delta t_i \ ,$$

$$\Delta T_g = \frac{g}{c^2}\sum_{i=1}^{N}(h_i - h_0)\,\Delta t_i \ ,$$

and

$$\Delta T_s = -\frac{\omega}{c^2}\sum_{i=1}^{N} R_i^2\cos^2\phi_i\,\Delta\lambda_i \ ,$$

respectively, where c is the speed of light, v is the velocity, g is the acceleration of gravity, $h - h_0$ is the relative height above the geoid, ω is the angular velocity of rotation of the Earth, R is the distance from the center of the Earth, ϕ is the latitude, and λ is the longitude over the path taken. Also, a range rate correction is applied to take into account the motion of the aircraft relative to the Earth's surface during the signal propagation time.

The results of the first flight test, shown in *Figure 2*, are representative of our results in general. The aircraft flew for nearly four hours at an altitude of 11 000 m (35 000 ft) at an average speed of 790 km/h (425 knots). The predicted (modeled) relativity effect agrees quite well with the measured clock offset data, except at the end of the flight when the TWTT radio link quality became poor.

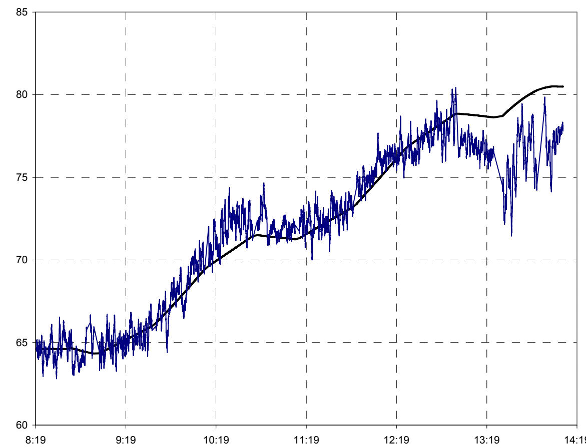


Figure 2. Comparison of measured and predicted net relativistic correction, ns (Flight Clock – Reference Clock).

VI. NASA FORMATION FLYING TESTBED

Future missions in NASA's Space Science Enterprise (SSE) will employ between a half dozen and three dozen spacecraft flown in precise formation to form a distributed interferometric instrument. Michelson interferometer concepts include the Terrestrial Planet Finder (TPF), the Submillimeter Probe of the Evolution of the Cosmic Structure (SPECS), and, further down the line, the Life Finder and Planet Imager concepts in the Origins theme (http://origins.jpl.nasa.gov). Several Fizeau interferometer imaging concepts are proceeding as well, including the Micro-arcsecond X-ray Imaging Mission (MAXIM) and the MAXIM pathfinder that will image the event horizon of a black hole (http://maxim.gsfc.nasa.gov). The Stellar Imager will perform asteroseismology and observe the internal dynamics of stars and (http://hires.gsfc.nasa.gov/~si). These missions all drive unprecedented levels of formation flying accuracy, both in the relative measurements and the relative control requirements. The control loops are closed through an inter-spacecraft communications network. In general, with varying separations between spacecraft, from hundreds of meters to a kilometer for the tightest tolerances, these requirements establish substantial problems in time transfer and time synchronization.

The Laser Interferometer Space Antenna (LISA) [6] has perhaps the most stringent metrology requirement (although no spacecraft-to-spacecraft formation control requirement). LISA is an interferometer formed by three heliocentric spacecraft and is designed to observe gravitational waves in a bandwidth of approximately 0.0001 Hz to 0.1 Hz. It involves the measurement of the separations of moving test masses within each spacecraft to a precision of one picometer over a distance of 5,000,000 km (1 part in 10^{23}).

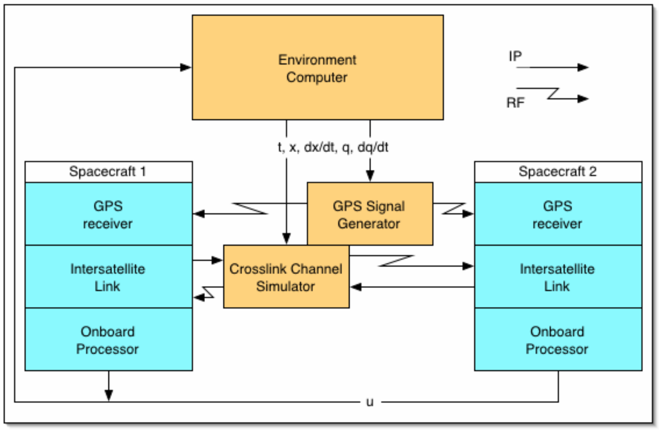


Figure 3. NASA Formation Flying Testbed (FFTB) configuration.

There are three main elements of the general formation flying interferometer problem that may set such requirements: (1) a direct need for time transfer or time synchronization for the virtual instrument due to a scientific requirement, (2) a derived requirement from the metrology system such that an accurate relative position measurement can be determined, and (3) a derived requirement from the formation control performance needed for phasing apertures from spacecraft to spacecraft. The requirements from elements (1) and (2) are rather straightforward to determine (or are directly specified from the science). The requirement established from (3) is much more complex. A realistic determination can only be made by integrating the actual communications and metrology systems in the loop with (possibly simulated) spacecraft and actual control laws implemented. These elements provide the focus for incorporating a high fidelity time simulator into the formation flying testbed (FFTB). A block diagram of the FFTB configuration is illustrated in *Figure 3*.

We must understand the effects of relative timing errors in the implementation of control laws through the communications network. How much performance error will we see in the aperture phasing for a given amount of timing error? How much can we tolerate and still be within the dynamic range to capture a fringe? The problem can be addressed in low fidelity as a multi-input, multi-output control system with arbitrary time delays strewn throughout. This low-fidelity assessment will not be able to tell us the feasibility of these control segments (or specifically controlling the path length) to the micron or nanometer level. Initially, we must work to establish those requirements and, eventually, we will use the testbed to establish a proper technology plan in relative navigation, formation control, timing, and communications.

VII. Conclusion

Time measurement and time dissemination is fundamental to space applications, such as on-board time registration, position determination and navigation, formation flying, imaging, interferometry, and theoretical science. To achieve the required precision of time transfer in a coherent, distributed network of clocks, theoretical algorithms must be developed in parallel to advances in clock technologies. These algorithms involve corrections for hardware biases, environmental factors, and relativistic effects.

We have created simulators to study the various factors, perform sensitivity analysis, and establish requirements. Work is continuing on these efforts, comprising research and documentation, experimental tests, and software development.

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